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NEUTRINO OSCILLATIONS IN THE FRAMEWORK OF THREE-GENERATION MIXINGS WITH MASS HIERARCHY

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Abstract

We have analyzed the results of reactor and accelerator neutrino oscillation experiments in the framework of a general model with mixing of three neutrino fields and a neutrino mass hierarchy that can accommodate the results of the solar neutrino experiments. It is shown that $\nu_\mu \leftrightarrows \nu_e$ oscillations with $0.6 \leq \Delta m^2 \leq 100\,\mathrm{eV}^2$ and amplitude larger than 2×10^{-3} are not compatible with the existing limits on neutrino oscillations if the non-diagonal elements of the mixing matrix $|U_{e3}|$ and $|U_{\mu 3}|$ are small. Thus, if the excess of electron events recently observed in the LSND experiment is due to $\nu_\mu \leftrightarrows \nu_e$ oscillations, the mixing in the lepton sector is basically different from the CKM mixing of quarks. If this type of mixing is realized in nature, the observation of $\nu_\mu \leftrightarrows \nu_e$ oscillations would not influence $\nu_\mu \leftrightarrows \nu_\tau$ oscillations that are being searched for in the CHORUS and NOMAD experiments.

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I. INTRODUCTION

The problem of neutrino mass and mixing is one of the major issues that confront us in neutrino physics at present. The search for the effects of neutrino mass is generally considered as one of the promising ways to probe new physics beyond the standard model. Many experiments are currently under way in different laboratories in order to investigate the effects of neutrino mixing and the nature of massive neutrinos (Dirac or Majorana?).

Important indications in favor of neutrino masses and mixing have been obtained in the solar neutrino experiments. The event rates measured in the all four solar neutrino experiments [1], which are sensitive to different parts of the solar neutrino spectrum, are significantly lower than the event rates predicted by the Standard Solar Model (see Ref. [2]). Moreover, a phenomenological analysis (see Ref. [3]) that does not depend on the predictions of the Standard Solar Model shows that the data from different experiments cannot be accommodated simultaneously if we assume that solar ν_e's are not transformed into other states.

Another indication in favor of non-zero neutrino mass comes from cosmology. The most plausible cosmological scenario of dark matter, which can describe the COBE data and large scale structures of the universe, is a mixture of cold dark matter with 20–30% of hot dark matter (see Ref. [4]). In this scenario it is conjectured that the neutrinos which form the hot dark matter have mass of the order of several eV.

New neutrino oscillation experiments searching for the effects of neutrino mass in the cosmological range are under way. The CHORUS [5] and NOMAD [6] experiments are looking for $\nu_\mu \to \nu_\tau$ oscillations and the KARMEN [7] and LSND [8] experiments are searching for $\nu_\mu \to \nu_e$ oscillations. It was reported recently [8] that an excess of 9 electron events with an expected background of 2.1 ± 0.3 events was observed in the LSND experiment. If these events are due to $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations, the average $\bar{\nu}_\mu \to \bar{\nu}_e$ transition probability is equal to $0.34^{+0.20}_{-0.18} \pm 0.07\%$.

We analyze here the results of the experiments that have searched for neutrino oscillations with reactor and accelerator neutrinos in the framework of a general model with mixing of three neutrino fields. We only assume that the neutrino masses satisfy a hierarchy that can accommodate the solar neutrino data. We show that a positive signal in the LSND experiment would mean that in the lepton sector there is no usual hierarchy of couplings among generations. We also discuss implications of a possible positive signal in the LSND experiment on $\nu_\mu \to \nu_\tau$ oscillation experiments.

II. THREE-GENERATIONS MIXING WITH NEUTRINO MASS HIERARCHY

We will discuss oscillations of terrestrial neutrinos in the framework of a scheme of mixing of three massive neutrino fields assuming only a hierarchy of neutrino masses.

According to the general theory of neutrino mixing (see, for example, Refs. [9,10]), the left-handed flavor neutrino fields $\nu_{\alpha L}$ are superpositions of the left-handed components of (Dirac or Majorana) massive neutrino fields ν_{kL}:

$$\nu_{\alpha L} = \sum_{k=1}^{n} U_{\alpha k} \nu_{kL} \ . \tag{2.1}$$

Here U is a unitary mixing matrix. From LEP data (see Ref. [11]) it follows that three neutrino flavors exist in nature. The number n of massive neutrinos depends on the scheme of neutrino mixing. In the case of a Dirac mass term, the total lepton number is conserved, massive neutrinos are Dirac particles and $n = 3$. In the case of a Majorana mass term, only left-handed flavor neutrino fields enter in the Lagrangian, massive neutrinos are Majorana particles and $n = 3$. In the most general case of a Dirac and Majorana mass term, massive neutrinos are Majorana particles and $n = 6$. In this last case there is a very attractive possibility for the generation of neutrino masses, i.e. the see-saw mechanism [12]. This mechanism is based on the assumption that the conservation of the total lepton number is violated at some large energy scale M. In this case, in the spectrum of Majorana masses there are three light masses m_k and three heavy masses, of the order of M. In the simplest see-saw model, the neutrino masses are given by $m_k \simeq m_{\mathrm{Fk}}^2/M$ ($k = 1, 2, 3$), where m_{Fk} is the mass of the up-quark or charged lepton in the k^{th} generation. The see-saw mechanism naturally explains the experimental fact that the neutrino masses are much smaller than those of other fundamental fermions.

We will consider here a scheme of mixings among three massive neutrino fields assuming that the neutrino masses satisfy the hierarchy relation

$$m_1 \ll m_2 \ll m_3 \tag{2.2}$$

which is suggested by the see-saw mechanism.

In the following we will not impose any theoretical constraints on the elements of the mixing matrix U. From Eq.(2.1) it follows that the state $|\nu_\alpha\rangle$ of a flavor neutrino with momentum p is given by

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U_{\alpha k}^{*} \, |\nu_k\rangle \ , \tag{2.3}$$

where $|\nu_k\rangle$ is the state of a neutrino with definite mass m_k. It is to be stressed that the state of a neutrino with a definite flavor is not a state with definite mass and the notion of mass of a flavor neutrino can have only an approximate meaning.

The amplitude of $\nu_\alpha \to \nu_\beta$ transitions can be written in the following form:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \mathrm{e}^{-\rangle \mathcal{E}_\infty \mathcal{L}} \left\{ \sum_{\|=\in}^{\ni} \mathcal{U}_{\beta\|} \left[\exp\left(-\rangle \frac{\cdot\Updownarrow^{\in}_{\|\infty} \mathcal{L}}{\in \surd} \right) - \infty \right] \mathcal{U}^{*}_{\alpha\|} + \delta_{\beta\alpha} \right\} \ . \tag{2.4}$$

Here L is the distance between the neutrino source and detector, $\Delta m_{k1}^2 \equiv m_k^2 - m_1^2$, p is the neutrino momentum and $E_k = \sqrt{p^2 + m_k^2} \simeq p + \dfrac{m_k^2}{2p}$.

We will assume that $\Delta m_{21}^2 \ll \Delta m_{31}^2$ is relevant for solar neutrinos, say $\Delta m_{21}^2 \simeq 5 \times 10^{-6}\,\mathrm{eV}^2$, as suggested by the MSW interpretation of the solar neutrino deficit. Thus, for

experiments with terrestrial neutrinos $\Delta m^2_{21} L/2p \ll 1$ and the probability of $\nu_\alpha \to \nu_\beta$ ($\bar\nu_\alpha \to \bar\nu_\beta$) transitions with $\beta \neq \alpha$ is given by (see, for example, Ref. [13])

$$P_{\nu_\alpha\to\nu_\beta} = P_{\bar\nu_\alpha\to\bar\nu_\beta} = \frac{1}{2}\, A_{\nu_\alpha;\nu_\beta} \left(1 - \cos\frac{\Delta m^2_{31}\, L}{2\, p}\right) , \tag{2.5}$$

where

$$A_{\nu_\alpha;\nu_\beta} = A_{\nu_\beta;\nu_\alpha} = 4\, |U_{\alpha 3}|^2\, |U_{\beta 3}|^2 \tag{2.6}$$

is the amplitude of $\nu_\alpha \leftrightarrows \nu_\beta$ ($\bar\nu_\alpha \leftrightarrows \bar\nu_\beta$) oscillations. Let us stress that Eq.(2.5) has the same form as the expression for the oscillation probability in the case of mixing between two generations. In this last case, the oscillation amplitude is equal to $\sin^2 2\vartheta$, where ϑ is the mixing angle.

The survival probability of ν_α ($\bar\nu_\alpha$) can be obtained from the unitarity constraint to be

$$P_{\nu_\alpha\to\nu_\alpha} = P_{\bar\nu_\alpha\to\bar\nu_\alpha} = 1 - \sum_{\beta\neq\alpha} P_{\nu_\alpha\to\nu_\beta} = 1 - \frac{1}{2}\, B_{\nu_\alpha;\nu_\alpha} \left(1 - \cos\frac{\Delta m^2_{31}\, L}{2\, p}\right) , \tag{2.7}$$

where the oscillation amplitude $B_{\nu_\alpha;\nu_\alpha}$ is given by

$$B_{\nu_\alpha;\nu_\alpha} = \sum_{\beta\neq\alpha} A_{\nu_\alpha;\nu_\beta} \; . \tag{2.8}$$

Using the unitarity of the mixing matrix we obtain

$$B_{\nu_\alpha;\nu_\alpha} = 4\, |U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right) \tag{2.9}$$

We wish to emphasize the following important features of terrestrial neutrino oscillations in the model under consideration:

1. All channels ($\nu_\mu \leftrightarrows \nu_e$, $\nu_\mu \leftrightarrows \nu_\tau$, $\nu_e \leftrightarrows \nu_\tau$) are open if all the elements $U_{\alpha 3}$ are different from zero.

2. The oscillations in all channels are characterized by the *same* oscillation length $L_{\mathrm{osc}} = 4\pi p/\Delta m^2_{31}$.

3. The equalities $P_{\nu_\alpha\to\nu_\beta} = P_{\bar\nu_\alpha\to\bar\nu_\beta}$ for $\alpha \neq \beta$ are satisfied even if CP is not conserved in the lepton sector.

4. The oscillation amplitudes $A_{\nu_\alpha;\nu_\beta}$ satisfy the following relation:

$$\left(\frac{A_{\nu_\mu;\nu_e} A_{\nu_\mu;\nu_\tau}}{A_{\nu_e;\nu_\tau}}\right)^{1/2} + \left(\frac{A_{\nu_\mu;\nu_e} A_{\nu_e;\nu_\tau}}{A_{\nu_\mu;\nu_\tau}}\right)^{1/2} + \left(\frac{A_{\nu_e;\nu_\tau} A_{\nu_\mu;\nu_\tau}}{A_{\nu_\mu;\nu_e}}\right)^{1/2} = 2 \; . \tag{2.10}$$

This relation follows from Eq.(2.6) and the unitarity of the mixing matrix.

III. DISCUSSION OF THE RESULTS OF NEUTRINO OSCILLATION EXPERIMENTS

In this section we will consider the results of the experiments that have searched for oscillations of terrestrial neutrinos in the framework of the scheme presented in Section II.

From the unitarity of the mixing matrix it follows that $|U_{\tau 3}|^2 = 1 - |U_{e3}|^2 - |U_{\mu 3}|^2$. Thus, oscillations of terrestrial neutrinos are characterized in this scheme by three positive parameters: $\Delta m^2_{31} \equiv \Delta m^2$, $|U_{e3}|^2$ and $|U_{\mu 3}|^2$.

Let us consider first the results of reactor and accelerator disappearance experiments in which $\bar\nu_e \to \bar\nu_x$ and $\nu_\mu \to \nu_x$ transitions were searched for. No indications in favor of neutrino oscillations were found in these experiments. We will use the exclusion plots which depict the limits obtained in the recent Bugey reactor experiment [14] and in the CDHS and CCFR accelerator experiments [15,16]. At fixed values of Δm^2, the allowed values of the amplitudes $B_{\nu_e;\nu_e}$ and $B_{\nu_\mu;\nu_\mu}$ are constrained by

$$\begin{aligned} & B_{\nu_e;\nu_e} \leq B^0_{\nu_e;\nu_e} \,, \\ & B_{\nu_\mu;\nu_\mu} \leq B^0_{\nu_\mu;\nu_\mu} \,. \end{aligned} \tag{3.1}$$

The values of $B^0_{\nu_e;\nu_e}$ and $B^0_{\nu_\mu;\nu_\mu}$ can be found from the corresponding exclusion curves.

From Eq.(2.9) the parameters $|U_{\alpha 3}|^2$ (with $\alpha = e, \mu$) can be expressed in terms of the amplitudes $B_{\nu_\alpha;\nu_\alpha}$ as

$$|U_{\alpha 3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - B_{\nu_\alpha;\nu_\alpha}}\right) . \tag{3.2}$$

Thus, from the negative results of reactor and accelerator disappearance experiments we see that the parameters $|U_{\alpha 3}|^2$ at fixed values of Δm^2 must satisfy one of the following inequalities:

$$\begin{aligned} & |U_{\alpha 3}|^2 \geq \tfrac{1}{2}\left(1 + \sqrt{1 - B^0_{\nu_\alpha;\nu_\alpha}}\right) \equiv a^{(+)}_\alpha \\ \text{or} \qquad & \\ & |U_{\alpha 3}|^2 \leq \tfrac{1}{2}\left(1 - \sqrt{1 - B^0_{\nu_\alpha;\nu_\alpha}}\right) \equiv a^{(-)}_\alpha \,. \end{aligned} \tag{3.3}$$

In Table I we present the values of $a^{(\pm)}_e$ and $a^{(\pm)}_\mu$ obtained from the negative results of the Bugey, CDHS and CCFR experiments for some values of Δm^2 in the interval $10^{-1}\,\mathrm{eV}^2 \leq \Delta \mathrm{m}^2 \leq 10^2\,\mathrm{eV}^2$, that covers the range where positive indications in favor of $\nu_\mu \leftrightarrows \nu_e$ oscillations were reported by the LSND experiment [8]. It can be seen from Table I that in the region of Δm^2 under consideration the values of $a^{(-)}_e$ and $a^{(-)}_\mu$ are small whereas the values of $a^{(+)}_e$ and $a^{(+)}_\mu$ are large (close to 1).

From Eq.(3.3) it follows that for any fixed value of Δm^2 we have four regions of possible values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. We will consider now all these regions.

I. The region of small $|U_{e3}|^2$ and $|U_{\mu 3}|^2$:

$$|U_{e3}|^2 \leq a^{(-)}_e \qquad \text{and} \qquad |U_{\mu 3}|^2 \leq a^{(-)}_\mu \tag{3.4}$$

The region of small values of the non-diagonal elements of the mixing matrix is the most interesting one from a theoretical point of view for only in this region the hierarchy of couplings among lepton generations can be realized.

To be specific, let us consider, as an example, the value $\Delta m^2 = 6\,\mathrm{eV}^2$. The allowed values of the parameters $|U_{e3}|^2$ and $|U_{\mu3}|^2$ are given by (see Table I)

$$|U_{e3}|^2 \leq 3.6 \times 10^{-2} \quad \text{and} \quad |U_{\mu3}|^2 \leq 2.0 \times 10^{-2} \,. \tag{3.5}$$

From these inequalities we obtain the following upper bound for the amplitude $A_{\nu_\mu;\nu_e}$ of $\nu_\mu \leftrightarrows \nu_e$ oscillations:

$$A_{\nu_\mu;\nu_e} \leq 2.9 \times 10^{-3} \,. \tag{3.6}$$

Additional restrictions for the allowed values of the amplitude $A_{\nu_\mu;\nu_e}$ can be obtained from the data of the experiments searching for $\nu_\mu \to \nu_\tau$ oscillations. In the region under consideration, from Eq.(2.6) we have

$$|U_{\mu3}|^2 \lesssim \tfrac{1}{4} A^0_{\nu_\mu;\nu_\tau} \,, \tag{3.7}$$

where $A^0_{\nu_\mu;\nu_\tau}$ is the upper bound of the amplitude of $\nu_\mu \leftrightarrows \nu_\tau$ oscillations. From the exclusion plot of the FNAL E531 experiment [17] which implies, at $\Delta m^2 = 6\,\mathrm{eV}^2$, $A^0_{\nu_\mu;\nu_\tau} = 2.5 \times 10^{-2}$, we obtain from Eq.(3.7) $|U_{\mu3}|^2 \lesssim 6 \times 10^{-3}$. This value, combined with the bound for $|U_{e3}|^2$ in Eq.(3.5), gives

$$A_{\nu_\mu;\nu_e} \lesssim 9 \times 10^{-4} \,. \tag{3.8}$$

This upper bound is smaller than the bounds found in the experiments searching for $\nu_\mu \to \nu_e$ transitions and the values of $A_{\nu_\mu;\nu_e}$ reported by the LSND experiment (see Fig.1). Thus, $\nu_\mu \leftrightarrows \nu_e$ oscillations with an amplitude larger than 9×10^{-4} at $\Delta m^2 = 6\,\mathrm{eV}^2$, are forbidden if both the parameters $|U_{e3}|^2$ and $|U_{\mu3}|^2$ are small.

In Fig.1 we have shown the upper bounds for the amplitude $A_{\nu_\mu;\nu_e}$ obtained from the exclusion plots of reactor and accelerator experiments for the case of small $|U_{e3}|^2$ and $|U_{\mu3}|^2$ in the range $10^{-1}\mathrm{eV}^2 \leq \Delta \mathrm{m}^2 \leq 10^2\,\mathrm{eV}^2$. The curve passing through the filled circles was obtained from the exclusion plots of the Bugey [14], CDHS [15] and CCFR [16] experiments. The curve passing through the open circles was obtained by combining the results of the Bugey experiment and the FNAL E531 [17] experiment, which was searching for $\nu_\mu \to \nu_\tau$ transitions, as explained above for the case of $\Delta m^2 = 6\,\mathrm{eV}^2$. In Fig.1 we have also plotted parts of the exclusion curves obtained by the KARMEN [7] (dash-dotted line) and BNL E776 [18] (dotted line) experiments which were searching for $\nu_\mu \to \nu_e$ transitions. Finally, taking into account that $A_{\nu_\mu;\nu_e} \leq B_{\nu_e;\nu_e}$ (see Eq.(2.8)), we also plotted in Fig.1 the exclusion curve for $B_{\nu_e;\nu_e}$ found in the Bugey experiment (dashed line).

It can be seen from Fig.1 that in the region of Δm^2 under consideration the results of reactor $\bar\nu_e$ and accelerator ν_μ disappearance experiments combined with those of

$\nu_\mu \to \nu_\tau$ appearance experiments provide more severe restrictions on the allowed values of the amplitude $A_{\nu_\mu;\nu_e}$ than the results of direct $\nu_\mu \to \nu_e$ appearance experiments. If any experiment searching for $\nu_\mu \leftrightarrows \nu_e$ oscillations finds, for Δm^2 in the range under consideration, an amplitude $A_{\nu_\mu;\nu_e}$ larger than the upper bound presented in Fig.1, it would mean that the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ cannot be both small, i.e. there is no natural hierarchy of generation couplings in the lepton sector. Such would be the case if the LSND result is confirmed, as it is seen from Fig.1, where the LSND allowed region is limited by the two thick solid lines.

The new experiments CHORUS [5] and NOMAD [6] searching for $\nu_\mu \to \nu_\tau$ transitions are under way at CERN. In Fig.1 we have also plotted the range of $A_{\nu_\mu;\nu_e}$ that could be explored when the projected sensitivity of the CHORUS and NOMAD experiments is reached (taking into account the bounds on $|U_{e3}|^2$ obtained from the Bugey experiment).

II. The region of large $|U_{e3}|^2$ and $|U_{\mu 3}|^2$:

$$|U_{e3}|^2 \geq a_e^{(+)} \qquad \text{and} \qquad |U_{\mu 3}|^2 \geq a_\mu^{(+)}. \tag{3.9}$$

The unitarity of the mixing matrix implies that $|U_{e3}|^2 + |U_{\mu 3}|^2 \leq 1$. It can be seen from Table I that in the range of Δm^2 under consideration the inequality $a_e^{(+)} + a_\mu^{(+)} > 1$ always holds. Therefore, the values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ cannot be both large, implying that this region is ruled out.

III. The region of large $|U_{e3}|^2$ and small $|U_{\mu 3}|^2$:

$$|U_{e3}|^2 \geq a_e^{(+)} \qquad \text{and} \qquad |U_{\mu 3}|^2 \leq a_\mu^{(-)} \tag{3.10}$$

If the neutrino masses satisfy the hierarchy relation (2.2), the survival probability of the solar neutrinos is given by [19]

$$P_{\nu_e \to \nu_e} = \left(1 - |U_{e3}|^2\right)^2 P^{(1,2)}_{\nu_e \to \nu_e} + |U_{e3}|^4 \ , \tag{3.11}$$

where $P^{(1,2)}_{\nu_e \to \nu_e}$ is the survival probability due to the mixing between the first and the second generations. Using the values of $a_e^{(+)}$ listed in Table I, we find that $P_{\nu_e \to \nu_e} \geq 0.92$ for all values of the neutrino energy. This lower bound is not compatible with the data of solar neutrino experiments [1], including the data of GALLEX and SAGE which have shown less neutrino deficit than the Homestake and Kamiokande experiments. Therefore, the allowed values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ cannot be in this region.

IV. The region of small $|U_{e3}|^2$ and large $|U_{\mu 3}|^2$:

$$|U_{e3}|^2 \leq a_e^{(-)} \quad \text{and} \quad |U_{\mu 3}|^2 \geq a_\mu^{(+)} \ . \tag{3.12}$$

The allowed region for the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ is considerably narrowed by the bounds obtained in the experiments searching for $\nu_\mu \to \nu_\tau$ transitions. In fact, using relation (2.6) we obtain

$$|U_{\mu 3}|^2 = \frac{1}{2}\left(1 - |U_{e3}|^2 \pm \sqrt{\left(1 - |U_{e3}|^2\right)^2 - A_{\nu_\mu;\nu_\tau}}\right) \tag{3.13}$$

At fixed values of Δm^2, from the corresponding exclusion curves we obtain

$$A_{\nu_\mu;\nu_\tau} \leq A^0_{\nu_\mu;\nu_\tau} \ . \tag{3.14}$$

From Eqs.(3.13) and (3.14) we find

$$|U_{\mu 3}|^2 \geq \frac{1}{2}\left(1 - |U_{e3}|^2 + \sqrt{\left(1 - |U_{e3}|^2\right)^2 - A^0_{\nu_\mu;\nu_\tau}}\right) \tag{3.15}$$

Further, from the results of the experiments searching for $\nu_\mu \leftrightarrows \nu_e$ oscillations, at any fixed value of Δm^2 we have

$$|U_{e3}|^2 \leq \frac{A^0_{\nu_\mu;\nu_e}}{4\,|U_{\mu 3}|^2} \ , \tag{3.16}$$

where the values of $A^0_{\nu_\mu;\nu_e}$ can be obtained from the exclusion curves of the experiments searching for $\nu_\mu \to \nu_e$ transitions.

It is also useful to notice that in the region under consideration the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are related with the amplitudes $A_{\nu_\mu;\nu_e}$ and $A_{\nu_\mu;\nu_\tau}$ by the following relations:

$$|U_{e3}|^2 \simeq \tfrac{1}{4}\, A_{\nu_\mu;\nu_e} \tag{3.17}$$

$$|U_{\mu 3}|^2 \simeq 1 - \tfrac{1}{4}\left(A_{\nu_\mu;\nu_e} + A_{\nu_\mu;\nu_\tau}\right) \tag{3.18}$$

These relations are valid for small $|U_{e3}|^2$ and $1 - |U_{\mu 3}|^2$.

Let us consider as an example $\Delta m^2 = 6\,\mathrm{eV}^2$. In Fig.2 we plotted the allowed region for the values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. The region (3.12) is limited by the dash-dotted and dash-dot-dotted lines. The unitarity limit $|U_{e3}|^2 + |U_{\mu 3}|^2 \leq 1$ is given by the thick solid curve. The allowed regions are indicated by the arrows. Using the value $A^0_{\nu_\mu;\nu_\tau} = 2.5 \times 10^{-2}$ given by the exclusion curve of the FNAL E531 experiment [17], from Eq.(3.15) we obtained the limiting curve represented as a dotted line in Fig.2. Using the limits $A_{\nu_\mu;\nu_e} \leq 5.0 \times 10^{-3}$ and $A_{\nu_\mu;\nu_e} \leq 1.4 \times 10^{-2}$ given by the BNL E776 [18] and KARMEN [7] experiments, from Eq.(3.17) we obtained the two limiting boundaries that are represented by the solid vertical lines in Fig.2. The regions A and B are allowed by all experiments except LSND. In Fig.2 we have also shown the region allowed by the LSND experiment (between the two thick solid lines). If we take into account the LSND result, only the region B is allowed.

If the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ satisfy the inequalities (3.12), the results of disappearance experiments and the experiments searching for $\nu_\mu \leftrightarrows \nu_\tau$ oscillations do not provide any constraint on $\nu_\mu \leftrightarrows \nu_e$ oscillations. If, for example, $\nu_\mu \leftrightarrows \nu_\tau$ oscillations are found by the CHORUS and NOMAD experiments with an amplitude $A_{\nu_\mu;\nu_\tau}$, it would mean that the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are related by Eq.(3.13) (with the plus sign), but the amplitude $A_{\nu_\mu;\nu_e}$ of $\nu_\mu \leftrightarrows \nu_e$ oscillations would not be constrained by this result. On the other hand, the observation of $\nu_\mu \leftrightarrows \nu_e$ oscillations with an amplitude $A_{\nu_\mu;\nu_e}$ would allow us to determine $|U_{e3}|^2$ (see Eq.(3.17)). However, as it is seen from Eqs.(3.17) and (3.18) the amplitude $A_{\nu_\mu;\nu_\tau}$ of $\nu_\mu \leftrightarrows \nu_\tau$ oscillations would not be constrained by this result. The same considerations apply to other values of Δm^2.

Let us notice that if a positive signal is found in the experiments searching for $\nu_\mu \leftrightarrows \nu_e$ oscillations and in the experiments searching for $\nu_\mu \leftrightarrows \nu_\tau$ oscillations, the values of both parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ would be determined unambiguously.

As is well known, if massive neutrinos are Majorana particles, neutrinoless double beta decay is allowed. The matrix elements of this process are proportional to $\langle m \rangle = \sum_i U_{ei}^2 \, m_i$. If, for example, $\Delta m^2 = 6\,\mathrm{eV}^2$, from Fig.2 we find the upper bound $|\langle m \rangle| \lesssim 3 \times 10^{-3}\,\mathrm{eV}$, which is much less than the sensitivity reachable by future experiments (see Ref. [20]).

Thus, if the model with mixing of three generations of neutrinos and a neutrino mass hierarchy turns out to be correct, the observation of a positive signal in $\nu_\mu \leftrightarrows \nu_e$ oscillation experiments with an amplitude larger than the limit given in Fig.1, would imply that $|U_{\mu 3}|^2$ is large and $|U_{e3}|^2$ is small. This result would mean that the neutrino mixing matrix is basically different from the Cabibbo-Kobayashi-Maskawa mixing matrix of quarks. We have stressed already that in the case of neutrino mixing, a neutrino with a given flavor does not have a definite mass. The approximate notion of "mass" of a flavor neutrino can be introduced only if one of the corresponding elements of the mixing matrix is large. It is obvious that in the region I, where $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are both small, ν_τ is the neutrino with largest "mass". If the elements of the mixing matrix are found to be in the region IV, a very unusual situation in which ν_μ is the neutrino with the largest "mass" would be realized.

IV. CONCLUSIONS

We have analyzed the results of reactor and accelerator neutrino oscillation experiments in the framework of a general model with mixing of three neutrino generations and a natural neutrino mass hierarchy $m_1 \ll m_2 \ll m_3$. Our only assumption is that $\Delta m^2_{21} \equiv m_2^2 - m_1^2$ is in the range suitable for solving the solar neutrino problem. In this model the oscillations of terrestrial neutrinos are described by three parameters: $\Delta m^2 \equiv m_3^2 - m_1^2$, m_3 being the heaviest mass, and the squared moduli of two elements of the mixing matrix, $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. Using only the results of disappearance experiments, at any fixed value of Δm^2 in the range $10^{-1}\,\mathrm{eV}^2 \leq \Delta \mathrm{m}^2 \leq 10^2\,\mathrm{eV}^2$, we found four possible regions for the values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. We have shown that if both parameters are small, the limit on the amplitude $A_{\nu_\mu;\nu_e}$ of $\nu_\mu \leftrightarrows \nu_e$ oscillations that can be obtained from the results of

reactor and accelerator disappearance experiments and from the results of the experiments searching for $\nu_\mu \leftrightarrows \nu_\tau$ oscillations is much less than the limit on $A_{\nu_\mu;\nu_e}$ that was obtained from direct $\nu_\mu \leftrightarrows \nu_e$ appearance experiments. From our analysis it follows that if the LSND signal is confirmed, it would mean that the values of the parameters $|U_{e3}|^2$ and $|U_{\mu3}|^2$ cannot be both small, i.e. there is no natural hierarchy of couplings among generations in the lepton sector. The region with large $|U_{e3}|^2$ and $|U_{\mu3}|^2$ and the region with large $|U_{e3}|^2$ and small $|U_{\mu3}|^2$ are excluded by the unitarity condition and by the results of the solar neutrino experiments, respectively. The region with small $|U_{e3}|^2$ and large $|U_{\mu3}|^2$ can accommodate the LSND signal. If this solution is realized in nature, the LSND signal would not influence $\nu_\mu \leftrightarrows \nu_\tau$ oscillations that are being searched for in the CHORUS and NOMAD experiments.

We have not discussed in this paper the atmospheric neutrino anomaly [21]. If future experiments, including long-baseline experiments (see, for example, Ref. [22]), confirm the existence of this anomaly and the result of the LSND experiment is confirmed also, three different Δm^2 scales would be required for the explanation of all these effects and the data of the solar neutrino experiments. In this case it would be necessary to assume the existence of an additional sterile neutrino state besides the three active flavor neutrino states. The discussion of models with three active and sterile states is out of the scope of this paper. Some models of this type have been recently considered [23].

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TABLES

TABLE I. Values of the upper and lower bounds $a_\alpha^{(-)}$ and $a_\alpha^{(+)}$ ($\alpha = e, \mu$) for the parameter $|U_{\alpha 3}|^2$ found from the results of reactor and accelerator disappearance experiments for some values of Δm^2 in the range $10^{-1} \leq \Delta m^2 \leq 10^2\,\mathrm{eV}^2$.

$\Delta m^2\,(\mathrm{eV})^2$	$a_e^{(-)}$	$a_e^{(+)}$	$a_\mu^{(-)}$	$a_\mu^{(+)}$
0.1	0.0096	0.9904	—	—
0.2	0.0083	0.9917	—	—
0.3	0.0088	0.9912	0.28	0.72
0.4	0.0083	0.9917	0.15	0.85
0.5	0.0078	0.9922	0.088	0.912
0.6	0.0050	0.9950	0.061	0.939
0.7	0.0065	0.9935	0.047	0.953
0.8	0.011	0.989	0.039	0.961
0.9	0.016	0.984	0.034	0.966
1	0.011	0.989	0.028	0.972
2	0.016	0.984	0.015	0.985
3	0.039	0.961	0.015	0.985
4	0.042	0.958	0.015	0.985
5	0.036	0.964	0.018	0.982
6	0.036	0.964	0.020	0.980
7	0.036	0.964	0.022	0.978
8	0.039	0.961	0.023	0.977
9	0.036	0.964	0.026	0.974
10	0.039	0.961	0.028	0.972
20	0.039	0.961	0.067	0.933
30	0.039	0.961	0.036	0.964
40	0.039	0.961	0.034	0.966
50	0.039	0.961	0.028	0.972
60	0.039	0.961	0.018	0.982
70	0.039	0.961	0.013	0.987
80	0.039	0.961	0.0076	0.9924
90	0.039	0.961	0.0076	0.9924
100	0.039	0.961	0.0076	0.9924

FIGURES

FIG. 1. The allowed regions in the Δm^2–$A_{\nu_\mu;\nu_e}$ plane obtained from the results of disappearance reactor and accelerator experiments and $\nu_\mu \leftrightarrows \nu_\tau$ appearance experiments for small values of $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. $A_{\nu_\mu;\nu_e}$ is the amplitude of $\nu_\mu \leftrightarrows \nu_e$ oscillations. The results from the BNL E776 and KARMEN $\nu_\mu \leftrightarrows \nu_e$ appearance experiments are also drawn. The region allowed by the LSND experiment is limited by the two thick solid curves.

FIG. 2. The allowed region for the values of the parameters $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ for $\Delta m^2 = 6\,\mathrm{eV}^2$ in the region of small $|U_{e3}|^2$ and large $|U_{\mu 3}|^2$. The region allowed by the LSND experiment is limited by the two thick solid lines.

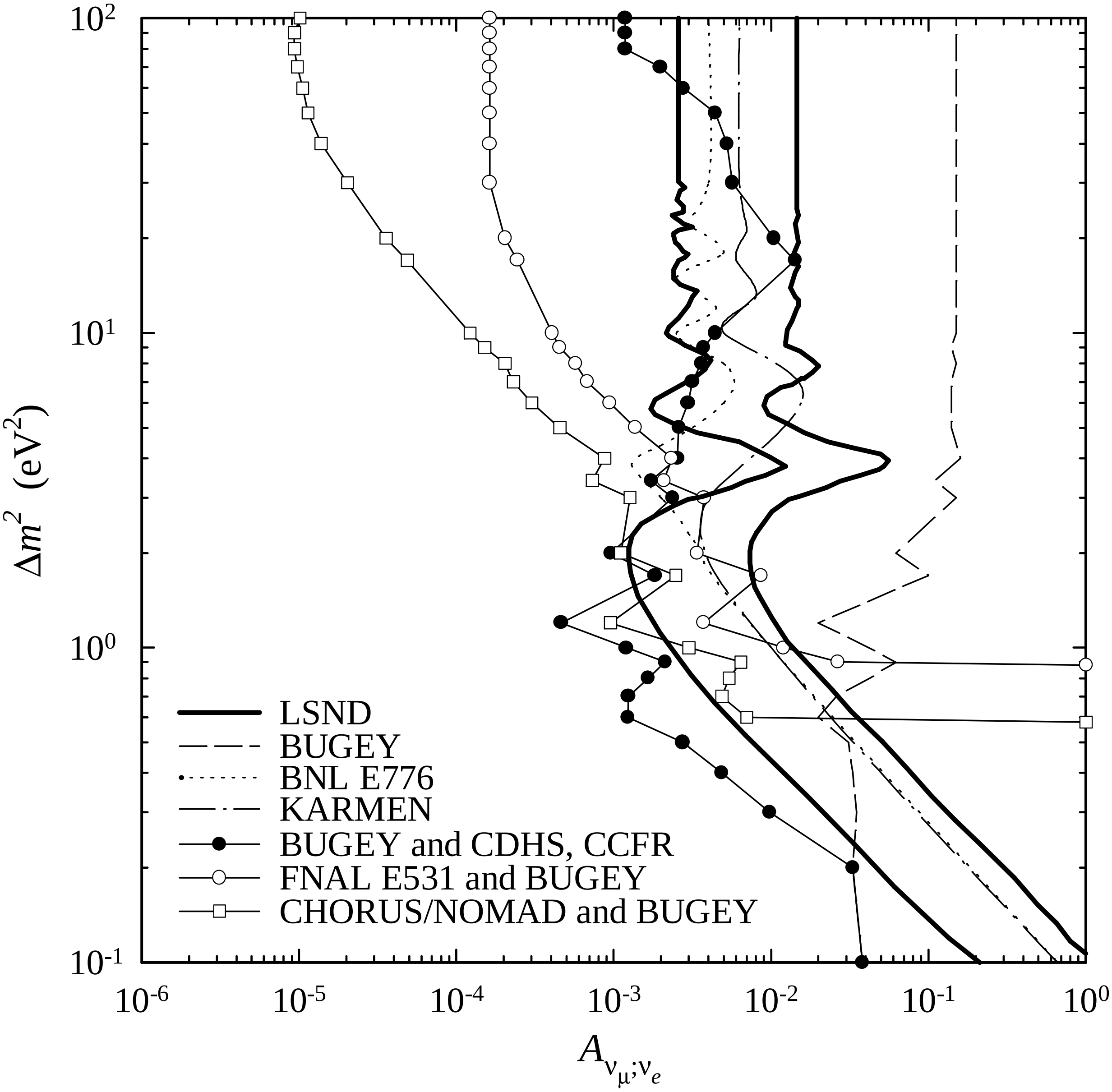

FIG. 1. The allowed regions in the Δm^2–$A_{\nu_\mu;\nu_e}$ plane obtained from the results of disappearance reactor and accelerator experiments and $\nu_\mu \leftrightarrows \nu_\tau$ appearance experiments for small values of $|U_{e3}|^2$ and $|U_{\mu 3}|^2$. $A_{\nu_\mu;\nu_e}$ is the amplitude of $\nu_\mu \leftrightarrows \nu_e$ oscillations. The results from the BNL E776 and KARMEN $\nu_\mu \leftrightarrows \nu_e$ appearance experiments are also drawn. The region allowed by the LSND experiment is limited by the two thick solid curves.

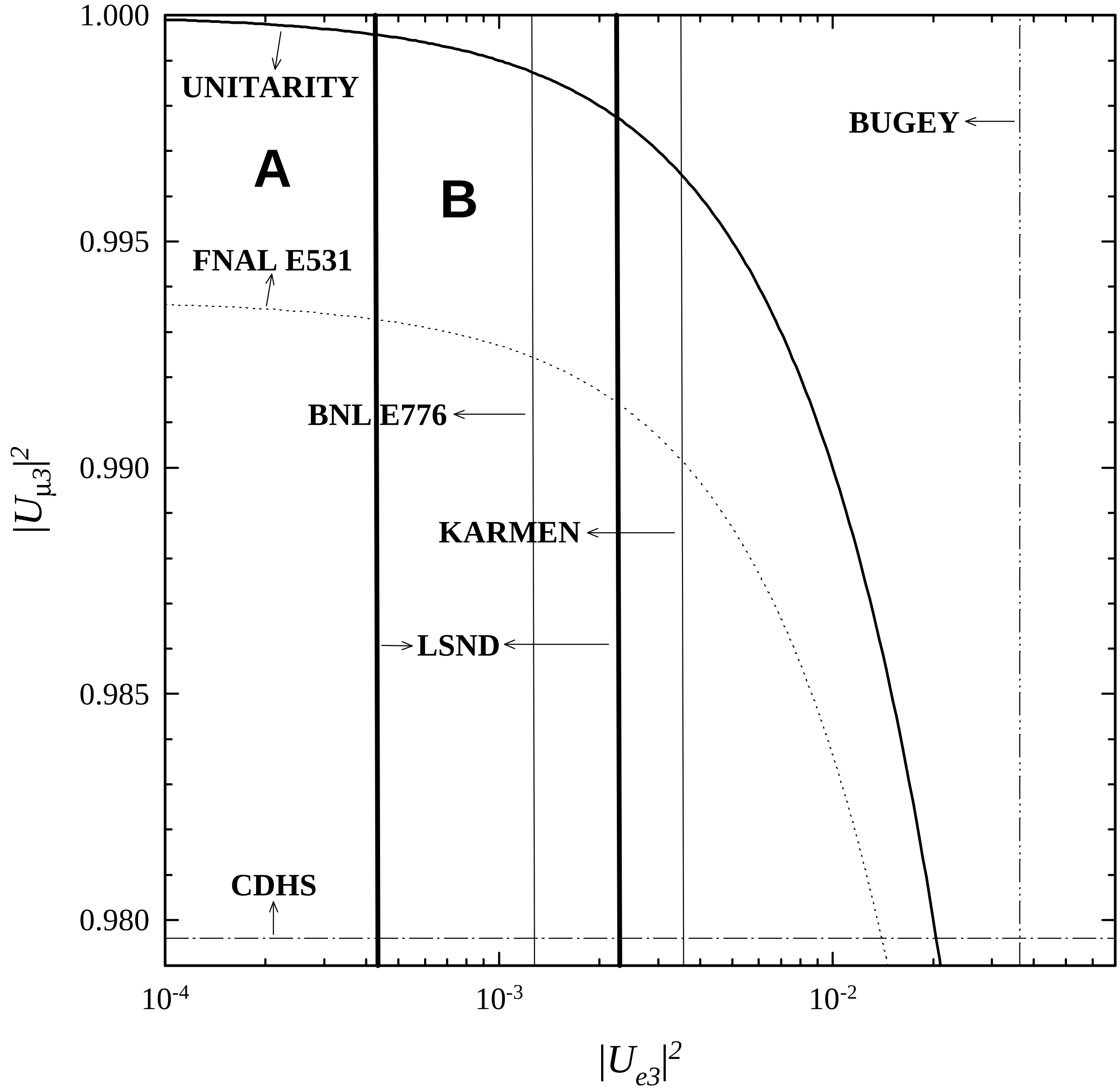

FIG. 2. The allowed region for the values of the parameters $|U_{e3}|^2$ and $|U_{\mu3}|^2$ for $\Delta m^2 = 6\,\mathrm{eV}^2$ in the region of small $|U_{e3}|^2$ and large $|U_{\mu3}|^2$. The region allowed by the LSND experiment is limited by the two thick solid lines.